PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Quest for a Three-Dimensional Theory of Ship-Wave Interactions [and Discussion]

J. N. Newman, W. C. Webster, G. X. Wu, A. E. Mynett, D. Faulkner and G. Victory

Phil. Trans. R. Soc. Lond. A 1991 **334**, 213-227
doi: 10.1098/rsta.1991.0009

The quest for a three-dimensional theory of ship–wave interactions

By J. N. Newman

Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, U.S.A.

The radiation and diffraction of water waves by ships can be analysed in classical terms from potential theory. The linearized formulation is well studied, but robust numerical implementations have been achieved only in cases where the vessel is stationary or oscillating about a fixed mean position. Slender-body approximations have been used to rationalize and extend the strip theory of ship motions, providing analytic solutions and guidance in the development of more general numerical methods.

The governing equations are reviewed, with emphasis on the interactions between the steady-state velocity field due to the ship's forward translation and the perturbations due to its unsteady motions in waves. Recent computations based on the boundary-integral-equation method are described, and encouraging results are noted. There is growing evidence that the influence of the steady-state velocity field is important, and the degree of completeness required to account for the steady field depends on the fullness of the ship. Benchmark computations are needed to test theories and computer programs without the uncertainty inherent in experimental comparisons.

1. Introduction

The principal wave loads and their effect on ships can be described by inviscid incompressible hydrodynamics, and hence by potential theory. (The notable exception is rolling motion, where viscous damping is significant.) The underlying boundary-value problem was known to Kelvin, Michell, Lamb and Havelock. A logical corollary might be that this field is computationally mature, but such a statement applies only in the limited context of small unsteady motions and no substantial forward velocity of the ship.

The restriction to wave motions of small amplitude, and correspondingly small motions of the vessel, is acceptable for most purposes despite the obvious nonlinear features of ocean waves and severe ship motions. Indeed, linearization is the foundation for spectral analysis, and thus for the useful synthesis of engineering predictions based on experiments as well as theory. On the other hand, the restriction to zero translational velocity, although appropriate for important special problems involving offshore platforms, is unacceptable for most ship-motion predictions.

In the absence of a practical three-dimensional approach, the quasi two-dimensional strip theory of ship motions has been used almost universally. This theory is conceptually and computationally simple: the flow at each transverse

Phil. Trans. R. Soc. Lond. A (1991) **334**, 213–227
Printed in Great Britain

section of the ship is governed by a two-dimensional Laplace equation and by a free-surface boundary condition which ignores convective acceleration. The strip theory can be derived rationally only in the asymptotic limit of short wavelengths relative to the ship length. Concerted efforts have been made to generalize this approach, based on slender-body approximations. Current research work has shifted from this more analytic objective to the development of numerical solutions of the three-dimensional flow field.

Why is the three-dimensional problem so difficult to solve, and why has the slender-body approach not been more fruitful? Where do we stand today in attempts to overcome the difficulties?

2. Theoretical formulations

The cartesian coordinate system $\boldsymbol{x}$ is defined to translate with constant velocity U in the $+x$-direction, and the ship oscillates about this moving reference frame. The z-axis is positive upwards with the origin in the undisturbed plane of the free surface. In most cases the unsteady motions of the ship, and corresponding velocity field of the fluid, are assumed small relative to the basic steady state. In the perturbation hierarchy any attempt to analyse these unsteady motions must be preceded by a solution of the basic steady-state flow, i.e. the 'wave-resistance problem' where the ship translates in otherwise calm water with constant velocity. In most cases simplifying assumptions or *ad hoc* procedures are used to avoid this complication.

With the assumptions of an ideal fluid and irrotational flow, the fluid velocity in the moving frame of reference is expressed as the gradient of the potential $U\bar{\phi}(\boldsymbol{x})+\varphi(\boldsymbol{x},t)$ where $U\bar{\phi}$ represents the steady base flow due to the ship's forward velocity and $\varphi(\boldsymbol{x},t)$ is the unsteady disturbance. Each potential is governed by Laplace's equation throughout the fluid domain. The important boundary conditions are on the submerged part of the ship hull 'body' surface S_{B} and on the free surface S_{F}. In addition a radiation condition must be imposed in the far field. These three conditions are reviewed here, following the more detailed description in Newman (1978).

(*a*) *Boundary condition on the body*

On S_{B} the normal component of the fluid and body velocities must be the same, where the latter is assumed known in terms of the six modes of rigid-body motion (surge, sway, heave; roll, pitch, yaw). If the amplitudes of these motions are sufficiently small, linear superposition is justified and six canonical potentials φ_j can be defined corresponding respectively to unit amplitudes of motion in each mode. Each of these potentials satisfies an inhomogeneous Neumann boundary condition on the instantaneous position of the body surface. These boundary conditions distinguish the solutions of the 'radiation problem', where the fluid motion is forced by the body oscillations.

Additional radiation modes can be defined to analyse the structural deflections of the hull, which occur as a result of the wave field and associated ship motions. In most cases these deflections are too small in amplitude to affect significantly the rigid-body modes. Thus the ship motions can be analysed without consideration of the structural deflections, which can be evaluated subsequently for the prescribed motions. In special circumstances this coupling must be accounted for, requiring that

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

the structural modes and impedance matrix be defined as a part of the motions analysis.

Separately the 'diffraction problem' is defined, with a prescribed incident-wave potential φ_0 plus a scattered component φ_7 caused by the body. The sum of these two potentials satisfies the condition of zero normal velocity on the body in its mean position. Alternatively, $\varphi_{7n} = -\varphi_{0n}$ on S_B and the boundary-value problem for the scattered potential is fundamentally similar to that of each radiation potential.

Subject to linear superposition, it is convenient to interpret the radiation and diffraction problems in physical terms with separate forcing. Thus there are no incident waves in the radiation problem, and no body motions in the diffraction problem.

To develop a consistent perturbation solution in the coordinate system $\boldsymbol{x}$ it is necessary to transfer the boundary condition from the instantaneous position of the body to its mean position. Following Timman & Newman (1962), with the notation introduced by Ogilvie & Tuck (1969), the result is

$$\varphi_{jn} = \mathrm{i}\omega n_j + U m_j, \tag{1}$$

where the unsteady motion is assumed periodic with time dependence $\mathrm{e}^{\mathrm{i}\omega t}$ and n_j denotes the jth component of the unit normal vector for the translational modes ($j = 1,2,3$) or the vector $(\boldsymbol{x} \times \boldsymbol{n})$ for the rotational modes ($j = 4,5,6$). (Subscripts other than the index j denote partial differentiation.) The extra terms with factors $(m_1, m_2, m_3) = -(\boldsymbol{n}\cdot\nabla)\,\boldsymbol{W}$ and $(m_4, m_5, m_6) = -(\boldsymbol{n}\cdot\nabla)\,(\boldsymbol{x}\times\boldsymbol{W})$ are due to the oscillatory motion of the body within the steady velocity field $\boldsymbol{W} = u\nabla\bar{\phi}$.

The so-called 'm-terms' in this boundary condition present special computational difficulties since second-derivatives of $\bar{\phi}$ are involved (Zhao & Faltinsen 1989). In strip theory, and in simplified three-dimensional computations, the assumption is made that $\boldsymbol{W} \approx -U\boldsymbol{i}$. In this case the only non-vanishing contributions are from the 'angle-of-attack' terms in (m_5, m_6), as in equation (2) below. For non-slender ships it is necessary to account for the steady disturbance, particularly near the bow and stern. The simplest approach is to use the 'double-body' solution for $\boldsymbol{W}$ corresponding to the zero-Froude-number limit of the steady problem where the free surface is a fixed horizontal plane. Even with this approximation careful numerical analysis is required to evaluate the m-terms.

Unlike the radiation problem, the forward velocity U does not affect the body boundary condition in the diffraction problem.

(*b*) *Boundary condition on the free surface*

If the unsteady potential φ is considered to be a perturbation of an arbitrary steady velocity field $\boldsymbol{W}$ the resulting linear boundary condition for φ on the free surface S_F contains variable coefficients, and must be applied on the exact steady free surface. To simplify this situation it is necessary to approximate $\boldsymbol{W}$. Various special cases are discussed below, in order of increasing complexity.

The simplest case is where $\boldsymbol{W} = 0$. The linearized unsteady potential then satisfies the classical free-surface condition $\varphi_{tt} + g\varphi_z = 0$ on the plane $z = 0$. In the frequency domain the simpler mixed condition $\omega^2\varphi - g\varphi_z = 0$ applies. If $U = 0$ this boundary condition is easily justified. The same boundary condition has been used in an *ad hoc* manner in strip theory; some justification follows from slender-body theory, where this condition applies in the inner domain near the body. An important consequence of the simple 'zero-speed' free surface condition is that analytic solutions for the

Green function exist which satisfy this boundary condition, and can be used to construct both analytic and numerical solutions (see Wehausen & Laitone 1960, §13; Newman 1990). In addition to forming a useful computational basis for vessels without forward velocity, this limiting case also has been used to develop a two-term perturbation analysis in powers of U which takes advantage of the simpler $U = 0$ Green function (Huijsmans & Hermans 1985; Hu & Eatock Taylor 1989; Nossen *et al.* 1989).

If $\boldsymbol{W} \approx -U\boldsymbol{i}$, the linear free-surface boundary condition is generalized by replacing the operator $\partial^2/\partial t^2$ by $(\partial/\partial t - U\partial/\partial x)^2$. This is appropriate if the ship is thin or slender, and more generally in the far field where three-dimensional attenuation reduces the magnitude of the ship's disturbance. A closed-form Green function can be used which satisfies this boundary condition either in the frequency domain (Wehausen & Laitone 1960, eq. 13.52), or in the time domain by an appropriate transformation of the transient Green function.

For non-slender ships it is necessary to account for the steady disturbance, and the situation is analogous to that of the boundary condition on S_B. The possibilities range from using the double-body solution for $\boldsymbol{W}$, or a linear solution of the steady free-surface problem with Kelvin waves, to the most general case where the steady flow is nonlinear. The free-surface condition for φ contains variable coefficients, and the analytic properties of the free-surface Green functions are of little help except in the far field.

(c) *The radiation condition*

In the frequency domain an appropriate radiation condition must be imposed in the far field, corresponding in the time domain to the requirement that the ship's disturbance should start from an initial state of rest. If $U = 0$ the radiation condition is equivalent to the simple requirement of outgoing circular waves at infinity. If $U \neq 0$ the far-field description is more complicated (Wehausen & Laitone 1960, fig. 3).

If the appropriate analytical form of the Green function is used, satisfying the linearized free-surface condition, the radiation condition is satisfied automatically by defining a suitable contour of integration. In strip theory the three-dimensional radiation condition is replaced by a conventional outgoing plane wave in two dimensions; this *ad hoc* substitution is justified only in the asymptotic sense for high frequencies. A more general 'unified' slender-body analysis (Newman 1978; Sclavounos 1984) reveals that a partial standing wave is appropriate in the matching domain, contrary to the more intuitive radiation condition of strip theory.

(d) *Numerical solution of the boundary-value problem*

Green's theorem can be used to reduce the unknown three-dimensional velocity potential to the solution of a two-dimensional integral equation on the boundary surface(s). Discretization of the boundary geometry and of the unknown solution yields a system of linear algebraic equations which can be solved numerically. For external flow problems this approach is particularly efficient if a suitable Green function can be used which reduces the computational domain to a single finite boundary surface, in this case S_B. Typically the boundary surface is approximated by a large number N of 'panels' or 'facets', with the same number of unknowns and equations based on collocation of the boundary conditions at each panel. The computations require N^2 evaluations of the coefficient matrix, in terms of the Green function and its derivatives, and subsequently the solution of the $N \times N$ linear system. In practice $N = O(1000)$ and substantial computational costs are associated with both the set-up and solution. Iterative solutions are practically essential for larger values of N.

This computational approach, often known as the 'boundary-integral-equation' or 'panel' method, has been applied to a variety of problems in inviscid fluid mechanics (Hess 1990). Alternative methods exist based on a finite-element or finite-difference representation of the fluid volume, but appear to be less effective due to the large computational domains required to describe external flows.

(e) *Wave loads and motions*

Once the velocity potential is known it is straightforward in principle to evaluate the pressure and integrate over the hull surface for the total force, moment, and structural loads. In practice the steady velocity field $\boldsymbol{W}$ introduces additional interactions at this stage, both in terms of products of the steady and unsteady velocities which must be accounted for in Bernoulli's equation, and transfer of the pressure between the instantaneous and mean positions of the hull surface.

In the frequency domain it is customary to express the components of the force in phase with the acceleration and velocity as the added mass and damping coefficients, respectively. As a consequence of the ship's unsteady motion within the steady velocity field there also exists an oscillatory restoring force in phase with the motions themselves, which augments the usual hydrostatic restoring force. This 'hydrodynamic restoring force' depends only on the Froude number. It is frequently overlooked or considered in experimental data to be a part of the added mass. The steady-state 'sinkage and trim' result from balancing the hydrodynamic and hydrostatic components of the restoring force and moment.

3. A simplified theory

A relatively simple three-dimensional analysis can be developed by combining slender-body approximations of the forward-speed effects with the numerical solution for $U = 0$. The result has some features of the strip theory, but with a three-dimensional solution of the velocity potential and pressure on the hull. The principal approximations are (1) to include only the free-stream velocity $-U\boldsymbol{i}$ in the m-terms of the body boundary condition, and (2) to neglect the steady velocity field completely in the free-surface condition. The first assumption can be justified if the ship hull is slender. Since the second assumption is *ad hoc* we postpone its use until the end of the analysis.

Consider the oscillatory heave (ξ_3) and pitch (ξ_5) motions of a slender ship, where the corresponding potentials satisfy the body boundary conditions

$$\varphi_{3n} = \mathrm{i}\omega n_3, \quad \varphi_{5n} = (-\mathrm{i}\omega x + U)\, n_3. \tag{2}$$

Here the approximation $n_5 \approx x n_3$ is used, based upon the assumption of slenderness. Denoting the solutions of (2) with $U = 0$ by a superscript $(^0)$, it follows that

$$\varphi_3 = \varphi_3^{(0)}, \quad \varphi_5 = \varphi_5^{(0)} + (U/\mathrm{i}\omega)\, \varphi_3^{(0)}. \tag{3}$$

Next we consider the differential vertical force $F'(x)$ acting upon the ship at the same station, where its surface intersects the plane of constant x in the contour $\Sigma(x, t)$.

From a momentum analysis which accounts for the movement of this contour (Newman 1977, eq. 102),

$$F' \approx -\rho\left(\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}\right)\int_{\Sigma}\phi n_3\,\mathrm{d}l, \tag{4}$$

where ϕ is the total perturbation potential due to the ship including the steady component but not the free stream, and the approximation neglects quadratic terms of order $(\phi)^2$.

The oscillatory component of F' includes a contribution from the interaction between the steady pressure field and the time dependence of Σ, as discussed at the end of §2. The remaining unsteady component of F' can be calculated directly from φ. Using (3), integrating over the length, and defining the integrals

$$F_j^{(0)} = \rho\omega^2\iint\varphi_j^{(0)}\,n_3\mathrm{d}S, \quad M_j^{(0)} = \rho\omega^2\iint(-x)\,\varphi_j^{(0)}\,n_3\,\mathrm{d}S, \tag{5}$$

it follows that the total vertical force F and pitch moment M acting on the ship are given in the forms

$$F = \xi_3 F_3^{(0)} + \xi_5(F_5^{(0)} + (U/\mathrm{i}\omega)\,F_3^{(0)}), \tag{6}$$

$$M = \xi_3\left(M_3^{(0)} - \frac{U}{\mathrm{i}\omega}F_3^{(0)}\right) + \xi_5\left(M_5^{(0)} + \frac{U^2}{\omega^2}F_3^{(0)} + \frac{U}{\mathrm{i}\omega}[F_5^{(0)} - M_3^{(0)}]\right). \tag{7}$$

Here integration by parts has been used in the longitudinal direction, and there are extra contributions from the ship's stern if it is not pointed. The contribution from the last pair of terms in (7) vanishes, if the cross-coupling coefficients are symmetric.

In the diffraction problem the unsteady potential includes the incident wave φ_0 and the scattered component φ_7. The Froude–Krylov force, due to φ_0, depends on the wavenumber and hence on the incident-wave frequency ω_0 in a fixed frame of reference. This component of the force is not dependent on U. Turning to the scattered component, the body-boundary condition $\varphi_{7n} = -\varphi_{0n}$ is independent of U and thus, in the notation used for the radiation solutions, $\varphi_7 = \varphi_7^{(0)}$. The differential scattering force can be derived from momentum conservation in the same manner as (4). The result is

$$F_7' \approx -\rho\left(\mathrm{i}\omega - U\frac{\partial}{\partial x}\right)\int_{\bar{\Sigma}}\varphi_7^{(0)}\,n_3\,\mathrm{d}l, \tag{8}$$

where $\bar{\Sigma}$ denotes the mean position of the ship cross section. Integrating along the length of the ship, it follows that

$$F_7 = F_7^{(0)}, \quad M_7 = M_7^{(0)} = (U/\mathrm{i}\omega)\,F_7^{(0)}, \tag{9}$$

where the definitions (5) are extended to apply with $j = 7$.

Structural loads can be determined directly from F', or by defining suitable higher-order mode shapes and integrating over the length of the ship.

The advantage of these expressions is that the effects of forward velocity in the body boundary condition and in the momentum integral for the differential force are accounted for explicitly, in terms of canonical potentials $\varphi_j^{(0)}$ which satisfy the simpler body boundary condition for $U = 0$. The only other role of the forward velocity is its implicit influence on these potentials via the free-surface boundary condition. A similar analysis by Salvesen *et al.* (1970), uses a variant of Stokes's theorem and direct pressure integration instead of the momentum relations (4) and (8). Salvesen *et al.* proceed from this point to invoke two-dimensional approximations for the components of the velocity potential, leading to a version of strip theory which has been used widely.

Following Inglis & Price (1980), one can avoid the two-dimensional strip-theory assumption and use instead the velocity potentials corresponding to the three-dimensional boundary-value problem with $U = 0$ in the free-surface boundary condition. The force and moment $F_j^{(0)}$, $M_j^{(0)}$ then can be evaluated from a three-dimensional panel code of the type generally associated with the analysis of offshore structures without forward velocity. If the scattering potential is considered in the same manner, the resulting solution for $\varphi_7^{(0)}$ depends on both ω_0 (via the body boundary condition) and ω (via the free-surface condition), but not otherwise on U. In this circumstance φ_0 and φ_7 satisfy different free-surface boundary conditions.

4. More complete numerical solutions

For computations where the free-surface boundary condition includes forward-speed effects, two distinct variants of the panel method have been used. These are discussed separately in the following sections.

(*a*) *Neumann–Kelvin approaches*

If the free-surface condition is linearized about a constant steady streaming flow $\boldsymbol{W} = -U\boldsymbol{i}$ (including the particularly simple special case where $U = 0$), the 'free-surface Green functions' defined in §2*b* can be used. Under these circumstances the computational domain is reduced to the surface of the ship hull, augmented in general by a line integral at the intersection of this surface with the plane of the undisturbed free surface. Thus the computational domain is minimized, at the expense of a more complicated Green function. Special algorithms and subroutines are desirable to ensure rapid and robust evaluation of the Green function, but progress in this direction is limited for the case where $U > 0$ (cf. Ohkusu & Iwashita 1989). An essential singularity occurs when the source and field point are in the free surface, and no appropriate algorithms exist which properly account for this singularity in panel methods.

Most existing works use *ad hoc* numerical integration to evaluate the Green function for $U > 0$, masking the singularity. Numerical results have been reported by Chang (1977), Inglis & Price (1982) and Guevel & Bougis (1982). Inglis & Price include the factors m_j in the hull boundary condition, and show a variety of results with varying degrees of correction for the forward velocity. In the most complete version a significant discrepancy exists in the vertical force distribution near the stern, and the authors suggest that viscous effects are responsible. Despite their impressive computational accomplishments, little practical implementation has followed from these works. Further progress may follow from the development of special algorithms for the Green function, but this is a substantially more complicated task in the case $U > 0$ and the essential singularity is an uncertain source of errors.

These difficulties can be circumvented by using a time-domain analysis, with the unsteady motions started from an initial state of rest. The corresponding Green function is equivalent to an impulsive point source at an arbitrary point in space. As time advances the steady forward velocity is represented by advancing the source points in the same manner used by Kelvin to analyse steady ship waves. For finite values of time the essential singularity does not exist, although in principle waves of monotonically decreasing wavelength will arise as time increases. The integral equation for the potential is solved at each time step by convolution of the solution for all previous times. To avoid redundant evaluations all of the previously computed values of the Green function must be stored. This requires substantial scratch memory (of order equal to the product of the time steps in the convolution and the square of the number of panels on the body). Extensive results are reported by (King *et al.* 1988) for a variety of mathematical and practical ship hulls; this work does not account for the ship's steady disturbance in evaluating the m-factors. In more recent work Beck & Magee (1990) discuss various numerical details, and compare the results using the undisturbed free stream against the double-body flow to evaluate the m-factors.

In the so-called 'body-nonlinear' time-domain analysis the linear free-surface condition and corresponding Green function are utilized, but the body boundary condition is satisfied on the exact surface S_B at each time step. This approach obviates the need to consider the troublesome m-factors, and the ship's motion may be prescribed arbitrarily, in principle. Lin & Yue (1990) have demonstrated significant progress with this approach, including geometry variations as the hull moves vertically in relation to the plane of the free surface. Since the geometry is non-stationary it is necessary to recalculate the values of the Green function (and its derivatives) at each time step, for all previous times required in the convolution integrals. For this approach to be feasible it is imperative to utilize fast algorithms for the Green function.

(b) *Rankine approaches*

Here the simpler 'Rankine' Green function $1/r$ is used, where r denotes the distance between the source and field points. In this case the free surface must be included in the computational domain of the integral equation, and radiation conditions in the far field must be considered explicitly. Advantages of this approach are (a) that the Green function is relatively simple to evaluate, and (b) that a variety of linear and nonlinear free-surface boundary conditions can be accommodated. This method was used originally by Gadd (1976) and Dawson (1977) for the computation of the steady wave-resistance problem, and has been developed extensively in that context. Recently the same method has been applied to the diffraction of incident waves by a moving submerged vessel by Bertram (1990), accounting for the 'exact' nonlinear steady flow field, and to the complete ship-motions problem by Nakos & Sclavounos (1990).

Nakos & Sclavounos consider only the double-body solution of the steady state, which is correct in the limit of zero forward velocity. Since the double-body flow also is trivially exact in the limit of zero body slenderness, their linearization about this state is considered to be consistent with both slow-ship and slender-ship approximations. The results are compared with the solution when only the steady uniform stream is accounted for, as in the Neumann–Kelvin problem, and substantial differences are shown for the cross-coupling coefficients. Most of this difference is associated with the body boundary condition, as compared with free-surface condition. Special efforts are made to develop a numerically consistent and robust algorithm for the free-surface discretization, with bi-quadratic B-splines used as the basis function for the unknown potential on each panel; this permits analytic

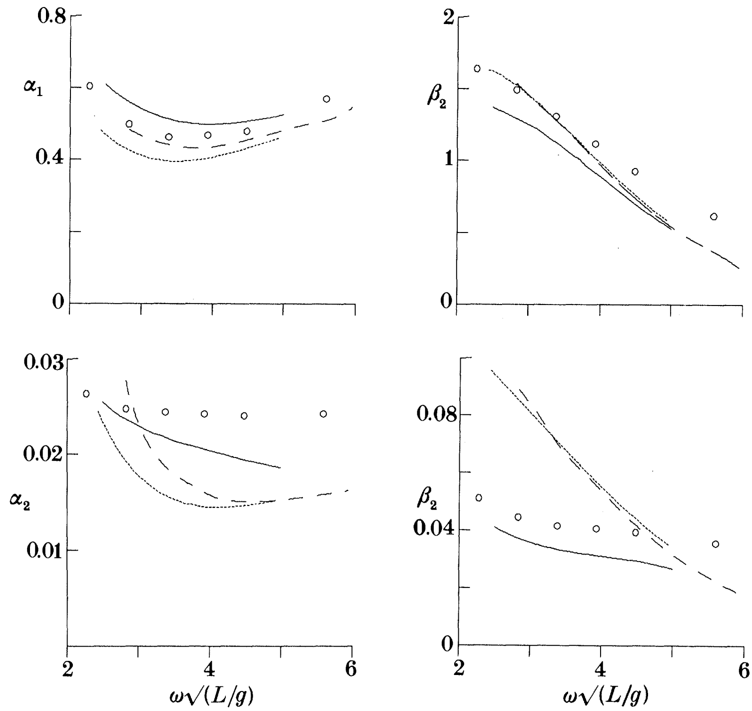

Figure 1. Added mass (a_{ij}) and damping (b_{ij}) coefficients for the Wigley hull in heave ($j = 3$) and pitch ($j = 5$). ○, Experiments; ---, three-dimensional approximate; ·····, strip theory; ——, Nakos & Sclavounos. $\alpha_1 = a_{33}/\rho\nabla$; $\alpha_2 = a_{55}/\rho\nabla L^2$; $\beta_1 = (b_{33}/\rho\nabla)\surd(L/g)$; $\beta_2 = (b_{55}/\rho\nabla L^2)\surd(L/g)$.

differentiation to be applied in the free-surface condition. The resulting solution is shown to be free of numerical damping, and numerical dispersion is controlled by suitably restricting the Froude number based on panel dimensions. A simple upstream radiation condition is invoked which limits the solution to moderate or large Froude numbers. The influence of the m-factors is computed by a variant of Stokes's theorem to avoid evaluating derivatives of the steady velocity field. Convergence with increasing numbers of panels is demonstrated, and satisfactory accuracy is obtained with $O(2000)$ total panels on the ship and free surface. Encouraging experimental comparisons are shown for the Series 60 ($C_B = 0.7$) hull at the Froude number $Fn = 0.2$, and for the modified Wigley hull at the $Fn = 0.3$. Selected results are reproduced in the following section.

5. Examples of numerical results

Special experiments have been conducted by Gerritsma (1988) with a mathematically defined 'Wigley hull' to provide a reliable basis for comparing theoretical results. This hull is defined by two-dimensional polynomials permitting accurate discretization and analysis by panel methods and other numerical techniques.

The results in figures 1–3 show comparisons of the experimental data with three theories. In order of increasing complexity these are (1) strip theory, (2) the simplified theory described in §3, where the same forward-speed corrections are used as in strip theory but the potentials are evaluated from a three-dimensional panel

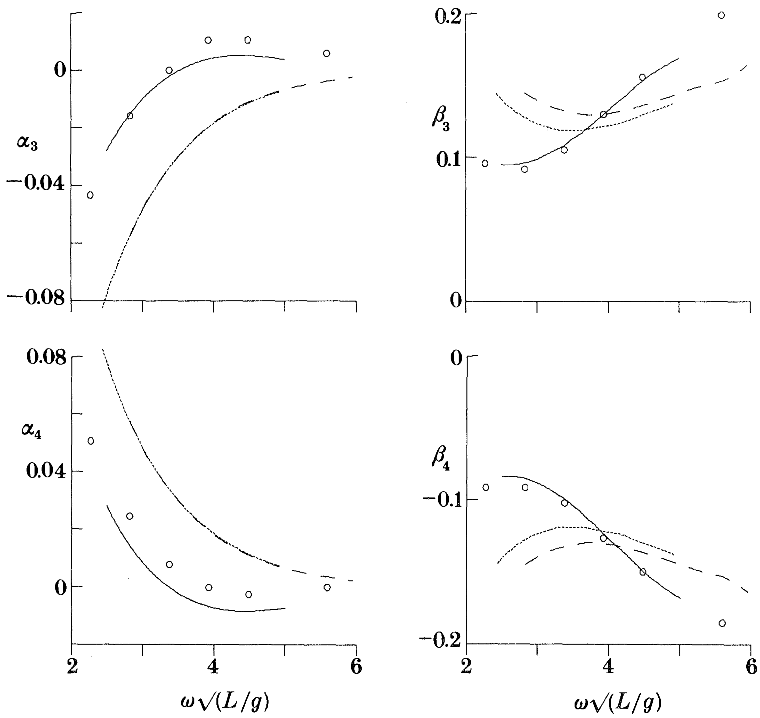

Figure 2. Cross-coupling coefficients between heave and pitch for the Wigley hull (see figure 1 for symbols). $\alpha_3 = a_{35}/\rho\nabla L$; $\alpha_4 = a_{53}/\rho\nabla L$; $\beta_3 = (b_{35}/\rho\nabla L)\times\surd(L/g)$; $\beta_4 = (b_{53}/\rho\nabla L)\ \surd(L/g)$.

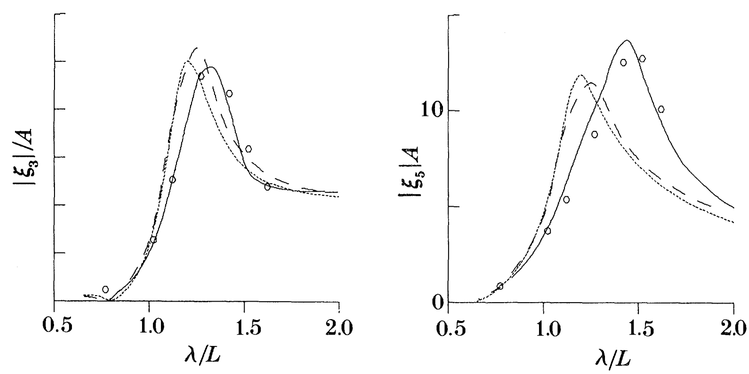

Figure 3. Non-dimensional heave and pitch motions of the Wigley hull in waves of amplitude A and wavelength λ (see figure 1 for symbols).

code, and (3) the more complete analysis of Nakos & Sclavounos described in §4. The agreement between the latter theory and the experimental data is substantially better than most previous work in this field.

For this relatively slender hull the differences between the strip theory and simplified three-dimensional theory are minor. Other results (not shown) for the Series 60 hull at a Froude number of 0.2 indicate somewhat better predictions from the simplified three-dimensional results compared with strip theory, but for both ships the theory of Nakos & Sclavounos clearly is superior. The most important differences appear in the cross-coupling coefficients (figure 2), where the slender-body forward-speed corrections in (6)–(7) appear to be exaggerated for the added mass and do not properly account for the frequency variation of the damping coefficients. In fact, if the forward-speed corrections are deleted completely there is no significant effect for the coefficients in figure 1, while for the cross-coupling coefficients in figure 2 the damping is worse but the added mass is improved! On this basis one must question the practical value of the slender-body forward-speed corrections, which have been so widely used in strip theories.

6. Conclusions

Vital connections exist between the three-dimensional description of unsteady ship motions and the theory of steady-state wave resistance. The later is a special case of the more general unsteady problem, but from the perturbation standpoint the steady problem must be solved first, and with more concern regarding nonlinear effects.

Similar analytical and computational methods have been applied to both the steady and unsteady problems. These include the thin-ship approach first advocated for the steady problem by J. H. Michell in 1898. (The unfulfilled promise of an unsteady sequel, noted by Michell in the closing of his famous paper is a disappointment to admirers of his work.) In the past 30 years parallel analytical developments have been applied to both the steady and unsteady problems based on slender-body theory and other geometric approximations. Parallel numerical solutions have been based on discretized boundary integral equations using both Rankine and free-surface Green functions.

Motivated by the commonality of these two problems, Faltinsen (1990) aptly states that 'To make further improvements in ship motion predictions at moderate and high Froude number it is felt that one first has to study the steady wave potential problem in more detail.'

On the other hand, some aspects of the steady and unsteady problems are different. The unsteady motions and structural loads in waves result primarily from the components of the pressure force acting on the hull perpendicular to its long axis, whereas the axial pressure force associated with the steady wave resistance is relatively small and thus difficult to evaluate with the same relative precision. Thus the low-Froude-number double-body approximation for $\bar{\phi}$ may be sufficiently accurate to describe the interaction between the steady and unsteady problems, despite the fact that the corresponding value of the wave resistance is zero. The results of Nakos & Sclavounos support this conjecture, for Froude numbers in the range 0.2–0.3. For high-speed vessels a more comprehensive solution of the steady velocity field may be required, or it may be practical to use a high-Froude-number approximation along the lines described by Zhao & Faltinsen (1990).

An unfortunate aspect of both the steady and unsteady problems is the absence of appropriate benchmarks with which to compare theoretical and numerical solutions. Experiments provide the only guidance in this respect, requiring judgements concerning the effects of viscosity and other practical considerations which should be addressed separately from the numerical implementation of the theory. If current research leads to such benchmarks in the near future, we can look forward to greater progress in this field.

Preparation of this paper was supported under the Accelerated Research Initiative on Nonlinear Ship Motions, sponsored by the Office of Naval Research.

References

Beck, R. F. & Magee, A. R. 1990 Time-domain analysis for predicting ship motions. *IU-TAM Symposium on the Dynamics of Marine Vehicles and Structures in Waves, Brunel University, London.*

Bertram, V. 1990 A Rankine source approach to forward speed diffraction problems. *Fifth International Workshop on Water Waves and Floating Bodies, Manchester.*

Chang, M. 1977 Computations of three-dimensional ship motions with forward speed. *Second International Conference on Numerical Ship Hydrodynamics, Berkeley*, pp. 124–135.

Dawson, C. W. 1977 A practical computer method for solving ship-wave problems. *Second International Conference on Numerical Ship Hydrodynamics, Berkeley*, pp. 30–38.

Faltinsen, O. M. 1990 *Sea loads on ships and offshore structures.* Cambridge University Press.

Gadd, G. E. 1976 A method for computing the flow and surface wave pattern around full forms. *Trans. R. Inst. nav. Architects* **118**, 207–216.

Gerritsma, J. 1988 Motions, wave loads, and added resistance in waves of two Wigley hull forms. *Rep. no.* 804, Technical University of Delft.

Guevel, P. & Bougis, J. 1982 Ship motions with forward speed in infinite depth. *Int. Shipbuilding Prog.* **29**, 103–117.

Hess, J. L. 1990 Panel methods in computational fluid dynamics. *A. Rev. Fluid Mech.* **22**, 255–274.

Hu, C. S. & Eatock Taylor, R. 1989 A small forward speed perturbation method for wave-body problems. *Fourth International Workshop on Water Waves and Floating Bodies, Øystese, Norway*, pp. 97–100.

Huijsmans, R. H. M. & Hermans, A. J. 1985 A fast algorithm for computation of 3-D ship motions at moderate forward speed. *Fourth International Conference on Numerical Ship Hydrodynamics, Washington*, pp. 124–135.

Inglis, R. B. & Price, W. G. 1980 Motions of ships in shallow water. *Trans. R. Inst. nav. Architects* **122**, 269–284.

Inglis, R. B. & Price, W. G. 1982 A three dimensional ship motion theory – comparison between theoretical predictions and experimental data of the hydrodynamic coefficients with forward speed. *Trans. R. Inst. nav. Architects* **124**, 141–157.

King, B. K., Beck, R. F. & Magee, A. R. 1988 Seakeeping calculations with forward speed using time-domain analysis. *17th Symposium on Naval Hydrodynamics, The Hague.*

Lin, W.-M. & Yue, D. K. P. 1990 Numerical solutions for large-amplitude ship motions in the time domain. *18th Symposium on Naval Hydrodynamics, Ann Arbor, Michigan.*

Nakos, D. & Sclavounos, P. D. 1990 Ship motions by a three-dimensional Rankine panel method. *18th Symposium on Naval Hydrodynamics, Ann Arbor, Michigan.*

Newman, J. N. 1977 *Marine Hydrodynamics.* Cambridge. Massachusetts: MIT Press.

Newman, J. N. 1978 The theory of ship motions. *Adv. appl. Mech.* **18**, 221–283.

Newman, J. N. 1990 The approximation of free-surface Green functions. *Retirement Meeting for Professor Fritz Ursell, University of Manchester.*

Nossen, J., Palm, E. & Grue, J. 1989 On the solution of the radiation and diffraction problems for a floating body with a small forward speed. *Fourth International Workshop on Water Waves and Floating Bodies, Øystese, Norway*, pp. 191–193.

Ogilvie, T. F. & Tuck, E. O. 1969 *A rational theory for ship motions – Part 1. Rep.* 013. Dept of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor.

Ohkusu, M. & Iwashita, H. 1989 Evaluation of the Green function for ship motions at forward speed and application to radiation and diffraction problems. *Fourth International Workshop on Water Waves and Floating Bodies, Øystese, Norway*, pp. 195–199.

Salvesen, N., Tuck, E. O. & Faltinsen, O. 1970 Ship motions and sea loads. *Trans. Soc. nav. Architects mar. Engrs* **78**, 250–287.

Sclavounos, P. D. 1984 The diffraction of free-surface waves by a slender ship. *J. Ship Res.* **28**, 29–47.

Timman, R. & Newman, J. N. 1962 The coupled damping coefficients of symmetric ship. *J. Ship Res.* **5**, 4, 34–55.

Wehausen, J. V. & Laitone, E. V. 1960 Surface waves. *Handbuch Physik* **9**, 445–778.

Zhao, R. & Faltinsen, O. 1989 A discussion of the m_j-terms in the wave-current body interaction problem. *Fourth International Workshop on Water Waves and Floating Bodies, Øystese, Norway*, pp. 275–279.

Zhao, R. & Faltinsen, O. 1990 Seakeeping of high-speed vessels. *Fifth International Workshop on Water Waves and Floating Bodies, Manchester*.

Discussion

W. C. Webster (*University of California, Berkeley, U.S.A.*). The six rigid-body motions can be decomposed into four parts, each with a distinct character. These are: (1) pitch and heave, (2) surge, (3) sway and yaw and (4) roll. Pitch and heave are most often used in the comparison with various theories, as Professor Newman does in his paper. Whereas it is clear that one can be somewhat optimistic about the ability to predict these two motions accurately within a decade, the situation with the other motions is unclear. What is the prognosis for the three sets of ‘forgotten’ motions?

J. N. Newman. Professor Webster refers to the ‘forgotten’ modes of ship motions. Roll is distinct from pitch and heave in respect to the timescale of resonance, degree of nonlinearity, and role of viscous damping. Surge, sway and yaw may be more amenable to theoretical descriptions, but the lack of hydrostatic restoration suggests the importance of low-frequency hydrodynamic effects and the treatment of the ship as a slender lifting surface. These motions have special relevance to aspects of safety such as broaching in quartering seas or capsizing in beam seas, and less importance in estimating structural loads.

G. X. Wu (*University College London, U.K.*). When $\tau < 0.25$ there are waves in front of the ship. The radiation condition assuming there is no wave in front of the ship has to be assessed. When the finite-element method was introduced to ship hydrodynamics in the early 1970s, the radiation condition was imposed on the boundary of localized finite elements. This was found extremely inefficient, and the coupled method was later developed. How far does the free surface have to go before the radiation condition can be imposed?

J. N. Newman. Responding to the computational issues raised by Dr Wu, although the radiation condition of Nakos & Sclavounos is well founded only for $\tau = \omega U/g > \frac{1}{4}$, their results indicate *ad hoc* validity for smaller values of this parameter, for small values of the frequency but not for small values of the Froude number. The singular region near $\tau = \frac{1}{4}$ is more problematic. The upstream limit of their computational domain is one-quarter of the ship length ahead of the bow.

A. E. Mynett. (*Delft Hydraulics, The Netherlands*). Regarding the accuracy of input wave data in relation to ship motion characteristics, it is important to distinguish between ship operations and ship design. For ship operations, knowledge of the actual ocean wave conditions is of great importance. At present, however, the accuracy of the ocean wave parameters is usually less than the accuracy of the ship's characteristics. On the other hand, for ship design, the emphasis is very often on intercomparison of various alternatives, keeping the input parameters fixed. Clearly, in this case the variability in the wave conditions is of less concern and the accuracy of determining the ship motion characteristics becomes of significance, indeed.

J. N. Newman. The discussants underscore many of the gaps between the mathematical description of linearized ship motions in an idealized wave environment and the ultimate description of wave-induced structural loads in an actual seaway of imprecise specification. The latter uncertainty is emphasized by Dr Mynett.

D. Faulkner. It seems that when structural analysts use linear strip theory for predicting ship vertical bending moments, from which for example short-term or long-term stresses are derived, that the theory consistently over-predicts these moments. Yet the same strip theory used to predict vertical plane ship motions of heave and pitch seems more often than not to slightly *under*-predict motions when compared with measured values in model tests and inferred from full-scale trials.

On the surface this would seem to present a paradox and I wonder why this is? Nonlinear effects such as different sag and hog buoyancy should presumably affect pitch motions, for example, as well as bending moments. Nonlinear buoyancy may not affect motions so directly as they do bending moments, but this still would not explain why motions are generally under-predicted whereas vertical bending seems to be consistently over-predicted. It is my understanding that the consistency of this over-prediction is actually made use of by a bias factor when choosing acceptable safety levels for RN frigate design.

J. N. Newman. Professor Faulkner notes the opposing errors of strip-theory predictions in the contexts of ship motions and of wave-induced structural loads. Most work on three-dimensional methods has concentrated on illustrative computations of the motions, and underlying force coefficients, as opposed to the structural loads. There is much evidence of the shortcomings of strip theory in the latter context, and more should be done to apply current three-dimensional computations to structural loads. The results will shed considerable light on the question of whether the overpredictions cited by Professor Faulkner are due to three-dimensional effects or nonlinearities.

G. Victory (*Surrey, U.K.*) After two very good photographs of a warship in rough weather, Professor Newman removed the effects of the very important part which such rough weather plays in ship–wave interactions in developing his theory. I have great difficulty in accepting that one can take away the effects of slamming, wave impacts on the structure or the flexing and severe vibrations in the structure caused by the interaction of ship and random wave impacts, and yet produce an acceptable theory which can be used in designing adequate strength into the ship.

J. N. Newman. Experimental evidence supports the linear analysis of ship motions, at least in head seas, despite the obvious nonlinear features of extreme conditions. A rational explanation is that the ship's dynamics are dominated by its very large mass, and thus it acts as a low-pass filter to suppress the effects of short-time high-frequency excitation. This is not to say that local effects of slamming, entrained water on deck, and structural deflections are insignificant, but only that these can be separated from the global description of the motions. More 'realistic' alternatives for engineering assessment such as full-scale observations, experiments, or nonlinear computations, all present complementary insight, but not without their own limitations.

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A